Optimal Operation of a Concentrator

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Abstract

The goal of this project is to find the optimum cycle time T, number of cycles per year *N*, and the heating element area *A*, that would minimize the total annual cost and corresponding total hours of operation of an industrial concentrator. The three unknown variables can be determined by manipulating the given equation for the instantaneous rate of evaporation, through integration by including various constraints and isolating one variable, A. Taking the partial derivative of the total annual cost equation by each variable and then equating them to 0, the minimum values can be found for T, N and A. The industrial concentrator, after calculations, will have a minimum total annual cost of $48,720.50 per year and operate for 993.8 hours per year to meet production goals of evaporating 1×10^6 kg of water per year for a given parameters in this problem.

Problem Statement

Many industrial processes are run in cycles – a cycle might run for *T* hours, then set up to run again. The number of cycles per year *N* and the cycle time *T* dictate the total hours of operation per year of the process.

Determine the optimum operating strategy for an industrial concentrator. To meet production goals, the concentrator must evaporate 1×10^6 kg of water every year. Specifically, the instantaneous rate of evaporation *Q* (in kg/hr) is: $Q=A\,(20-0.01\,t)$. Where *A* is the surface area of the heating element (in m²) and t is the time since the beginning of the current cycle in hours.

At the end of each cycle, there is a cost of $1000 to clean the heating element. When the cycle is in operation, there is a labor cost of $20/hr. The cost of the evaporator, prorated over its useful life, is $400 per year per m² of surface area. Cost of electricity to generate the heat is $0.004/kg of water evaporated. The optimal operating strategy is the one that minimizes the total annual cost. Please determine:

(a) The optimum cycle time T, number of cycles per year *N* and heating element surface area *A*.

(b) The corresponding annual cost and total hours per year of operation.

Mathematical Approach

When first approaching to solve this problem, we recognized that the given equation $Q=A\,(20-0.01\,t)$ is the instantaneous rate of evaporation, i.e., the derivative of evaporation *E* over time *t*. To get the amount of water evaporated in one cycle, the equation integrated from *0* to *T*:

$$E(T) = \int_0^T Q(t)\,dt = \int_0^T A\,(20-0.01\,t)\,dt$$

$$= \left[20\,A\,t - \frac{0.01At^2}{2}\right]_0^T \qquad (1)$$

The concentrator must evaporate 1×10^6kg of water per year, thus: $\left(20\,A\,T - \frac{0.01AT^2}{2}\right)N = 1\times10^6 \qquad (2)$

Now we can formulate a total cost equation:

$$C = \$1000\,N + \$20\,N\,T + \$400\,A + \$0.004\,(1\times10^6). \qquad (3)$$

Total Hours of Operation (*H*) of the concentrator:

$$H = NT \qquad (4)$$

Now we can solve for A in (3) and substitute that value for A in (4) so we have it in terms of N and T:

$$\left(20\,A\,T - \frac{0.01\,A\,T^2}{2}\right)N = 1\times10^6 \quad \Rightarrow \quad A = \frac{1\times10^6}{(20\,T-0.005T^2)\,N}$$

N

$$C(N,T) = 1{,}000\,N + 20\,N\,T + 400\,\frac{1\times10^6}{(20\,T-0.005T^2)\,N} + 4000$$

(6)

We can n... take the partial derivatives of C with respect to N and T a... ize *C* and isolate *N* ...

$$\frac{\partial C}{\partial N} = 1000 + 20\,T - \frac{4\times10^8}{(20\,T-0.005T^2)\,N^2} = 0$$

$$\frac{\partial C}{\partial T} = 20\,N - \frac{4\times10^8(20-0.01\,T)}{N\,(20\,T-0.005T^2)^2} = 0 \qquad (7)$$

$$N^2 = \frac{4\times10^8(20-0.01\,T)}{20\,(20\,T-0.005T^2)^2}.$$

With respect to T: (8)

Solving for *N*: (9)

$$20\,T - \frac{20\,T\,(20-0.005\,T)}{(20-0.01\,T)} = -1000$$

Substitute *N*² (10)

Solving (11) for T using a graphing utility there are two intercepts: *T = 400 or -500* (11)

Since T represents time in hours, the value should be a ... for N and substitute 400 in for T:

$$N^2 = \frac{400\times10^6(20-0.01\times400)}{20(20\times400-0.005\times400^2)^2} \quad \Rightarrow \quad N = 2.48 \text{ cycles/year}$$ 9)

(12)

Now knowing N and T, A = 55.9 m² from (5), H = 993.8 hours/year from (4) and the total annual cost from (3): C = $48,720.50

Discussion

The results of this project proved to be successful in that I was able to find the number of cycles (*N*), cycle time (*T*) and surface area of the heating element (*A*), that minimized the total annual cost of the concentrator. The results for the first two variables *N* = 2.4845 cycles per year and T = 400 hours were used to calculate *A*, where *A* = 55.9 m² represents the surface area of the heating element. Using these requirements for the concentrator it was calculated that the minimum total annual cost of the concentrator *C* = $48,720.50 and the total hours of operation *H* = 993.8 hours per year. Overall these results provide an optimum operating strategy for this industrial concentrator that will provide it to operate at top efficiency and be at the cheapest cost to the owner.

One option to consider is that N is the number of cycles per year and perhaps the concentrator could only run a full cycle as opposed to 2.4845 cycles. In that case one would have to round *N* to *N* = 3 cycles per year and use that new optimum *N* to calculate a new *T*, *A*, *C* and *H*. These results would provide the most efficient means of operation for an industrial concentrator that could only run an integer number of cycles. Although these new parameters do provide an interesting interpretation to the data, all of the original calculations in the mathematical solutions section are correct under the initial parameters of the project objective.

The results calculated in this project could be very useful to engineers dealing with industrial concentrators of any kind. In this problem the parameters happened to be that the concentrator had to evaporate 1×10^6 kg of water per year and there were several other costs such as a labor cost, cleaning cost, fixed cost of the evaporator and the cost of electricity power the heating element. Engineers in the field that have parameters similar to these and could use our calculations under their specifications to take a similar approach on achieving the optimum result of minimizing the total annual cost of a concentrator.

Conclusions

This project was successful because when solved correctly, it provided the number of cycles, cycle time and surface area of the heating element that minimized the total annual cost of the concentrator. This particular concentrator will run *2.48* cycles per year at *400* hours per one cycle. Thus the concentrator will run for *993.8* hours per year with a heating element having area an area of *55.9* m². Those values ensure a minimum total annual cost of *$48,720.50*. Overall these results provide an optimum operating strategy for this industrial concentrator that will provide it to operate at top efficiency.

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Appendix

Nomenclature

Symbol	Description	Units
N	Number of cycles per year	cycles/year
T	Cycle time	hrs
A	Surface area of heating element	m²
Q	Instantaneous evaporation rate	kg/hr
C	Total cost of operation	$/year
H	Total hours of operation	hours/year

Cost Information

Starting Parameter	Value
Amount of water to evaporate	1×10^6 kg/year
Cost of cleaning	$1,000/cycle
Cost of labor	$20/hour
Prorated cost of evaporator	$400/(m² · year)
Cost of electricity	$0.004/kg of water