Simplified Model of the Internal Combusiton Engine

Mechanical Engineering

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Introduction

This project sets out to replicate a given theoretical model. Using Euler's modified method for second order differential equations, with a set of initial conditions and limitations. A numerical simulation was then created according to these limitations:

- A sufficiently lightweight flywheel is present(nearly massless)
- A Scotch yoke linkage is used for a connecting rod
- Engine is operating on an Otto cycle using air as the working fluid

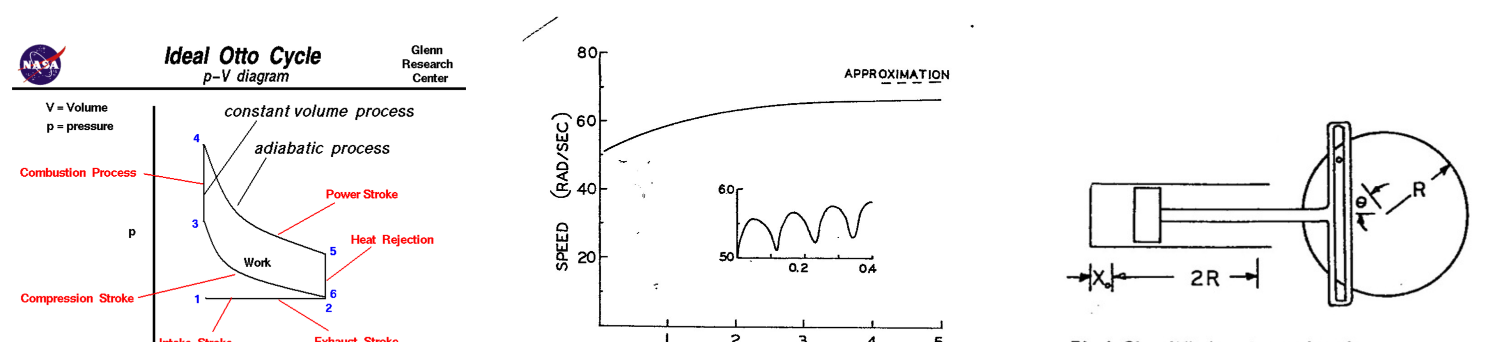


Fig 1. Simplified engine and cycle

Problem Statement

Recreate a manageable model of a two-stroke internal combustion engine to demonstrate possible fluctuations due to a varying distribution of power and load simplified by the use of a Scotch yoke linkage for the connecting rod (figure 1). Upon modeling the mechanical system, replicate the results provided by SC Kranc's journal entry to an approximation of $72s^{-1}$ average angular velocity.

Mathematical Description

The fundamental equation is derived by summing the moments of inertia on the flywheel assuming the connecting rod is massless (Kranc 343)(1):

$$I\ddot{\theta} = T_{piston} - T_{load}$$

Force on the piston's area A acts with a moment arm R sine of theta. Torque of the load is modified as it is proportional to the square of the angular velocity(2):

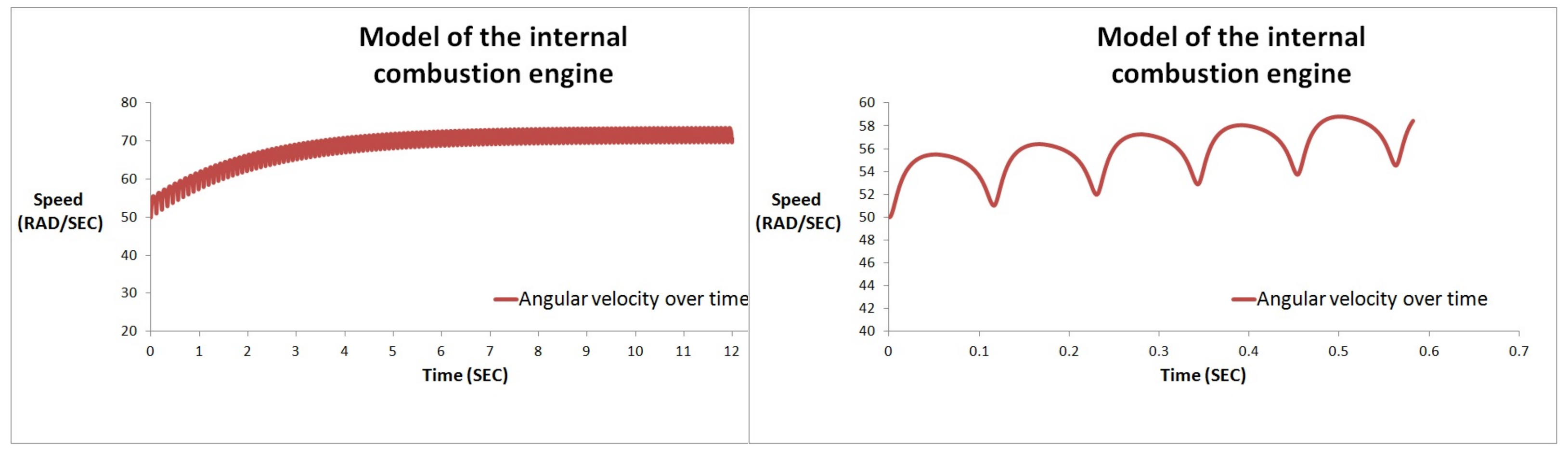
$$I\ddot{\theta} = PARsin\theta - C(\omega)^2$$

During the compression stroke, the polytropic law with exponent n accounts for the relationship between pressure, volume, and temperature. Thus, volume of the cylinder at any stroke is(3):

$$V = (R - Rcos\theta + x_o)A$$

Then, the fundamental equation (4) comes to light by uniting equation (2) and (3):

$$\omega' = \ddot{\theta} = \begin{cases} \omega' a = P_1 \left(\frac{R - Rcos\theta_1 + x_o}{R - Rcos\theta + x_o}\right)^n \frac{ARsin\theta}{I} - \frac{C\omega^2}{I} \\ and \\ \omega' b = P_3 \left(\frac{R - Rcos\theta_3 + x_o}{R - Rcos\theta + x_o}\right)^n \frac{ARsin\theta}{I} - \frac{C\omega^2}{I} \end{cases} \frac{for\ \pi<\theta<2\pi}{for\ 0<\theta<\pi}$$



()Primary chart left, zoomed in chart right.

Euler's Modified Method

Formula,

$$\left(\frac{df}{dt}\right)_{t=t_0} = f'(t_0) = \lim_{\Delta t \to 0} \frac{f(t_0 + \Delta t) - f(t_0)}{\Delta t} \qquad f(t_1) \approx f(t_0) + \Delta t \cdot f'(t_0)$$

Euler's method consists of a step-wise procedure using the differential equation to calculate derivatives at a particular time, that relates dependent and independent variables. Omega, theta and time are dependent variables in this case. To evaluate, we begin at 50s-1, pi, and 0.001 respectively.

$$\frac{d^2\theta}{dt^2} = \omega' \Rightarrow \frac{d\theta}{dt} = \omega \Rightarrow \theta' = \omega$$

Properly simplifying the second order differential equation, two first order equations were set up for our dependent variables,

$$\omega_{new} = \omega_{previous} + \Delta t \omega'_{previous} \qquad \theta_{new} = \theta_{previous} + \Delta t \theta'_{previous}$$

Discussion

After applying Euler's modified method to the differential equation the results were as desired yielding an average of $72s^{-1}$ as well as a spot on image of the graph provided in the journal entry. Within the 12,000 trials or 12 seconds using 0.001 intervals;

- 132 cycles or revolutions from 0 to 2π were taken
- A change in equation of ω' every 60 trials or 0.060 seconds approximately
- A change in cycle every 120 trials or 0.12 seconds approximately

Conclusions and Recommendations

The objective of this project was met by recreating a manageable model of a simplified internal combustion engine to a suitably small approximation using Euler's method for ordinary differential equations. For future reference, this project could be expanded by modifying the system;

- Recreate a four-stroke engine or reciprocating compressor
- Test the efficiency of the engine
- Change the initial conditions and limitations, noting any significant change

References

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