U.S. Oil Reserves and Peak Oil

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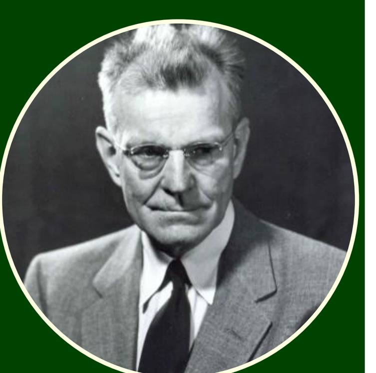
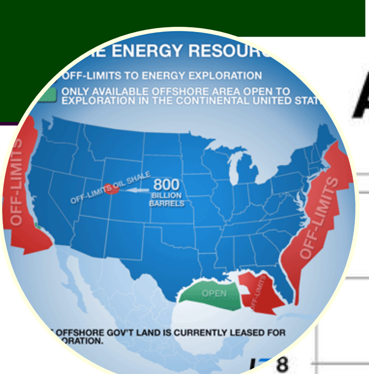
Abstract

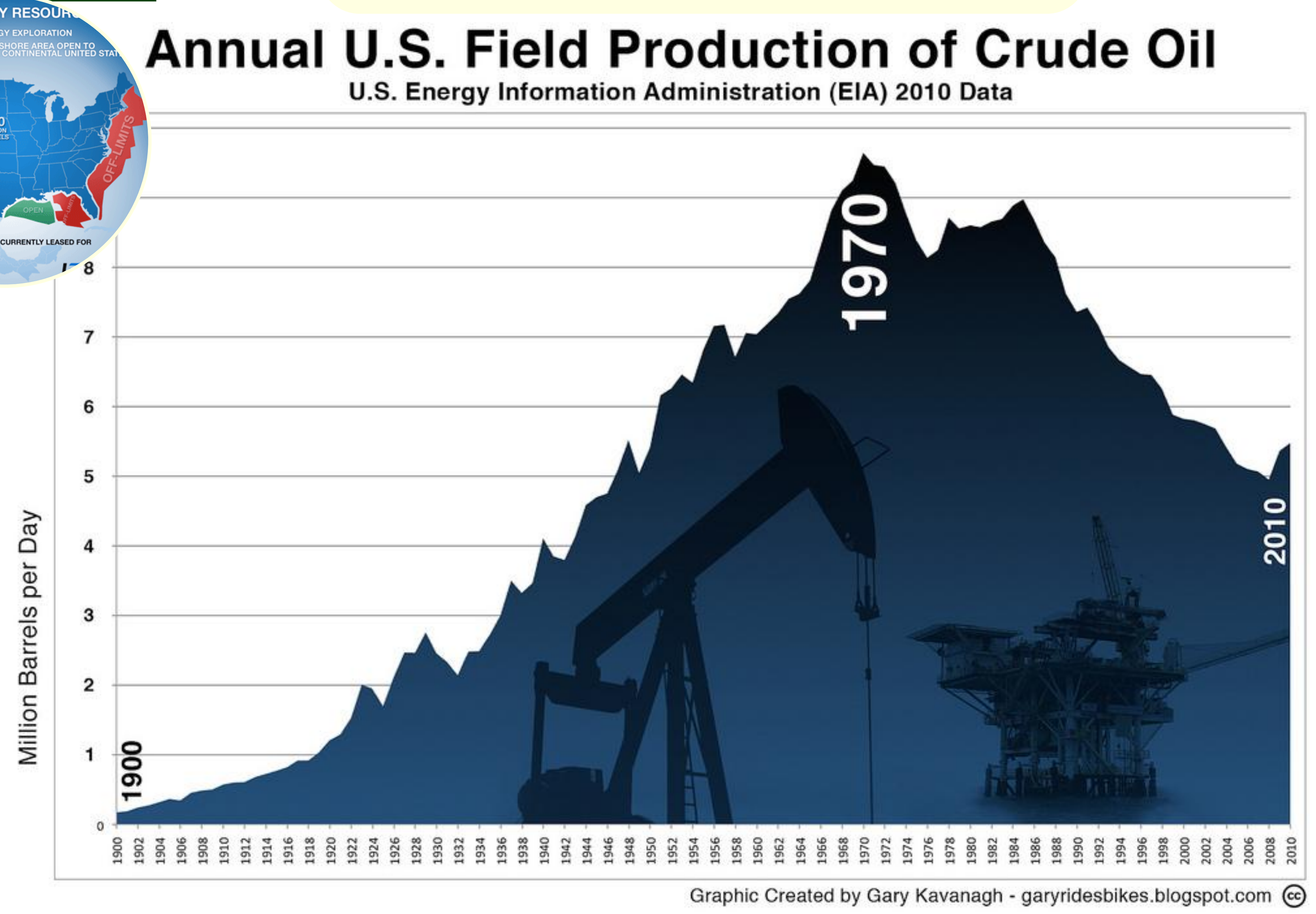
Based on the data of U.S. Field Production of Crude Oil from 1859 to 2010, this project provides an estimate of the US oil reserves by nonlinear regression. Through the creation of new fitting formula, the project calculates the year at which the U.S. production reached its peak, the estimated total amount of petroleum in the U.S., including the amount already recovered, as well as known and unknown reserves. The calculations of the project determine that the undiscovered U.S. oil reserves are about 31 billion barrels, which can supply for the oil industry for ten more years if they keep the same rate of production as current time.



Problem Statement

In 1956, M. King Hubbert, an American geologist, postulated that US oil reserves are finite and he predicted that the oil productions would reach a maximum in the early 1970s. Therefore, all of the total petroleum available is represented by the area under the U.S. Oil Production curve.



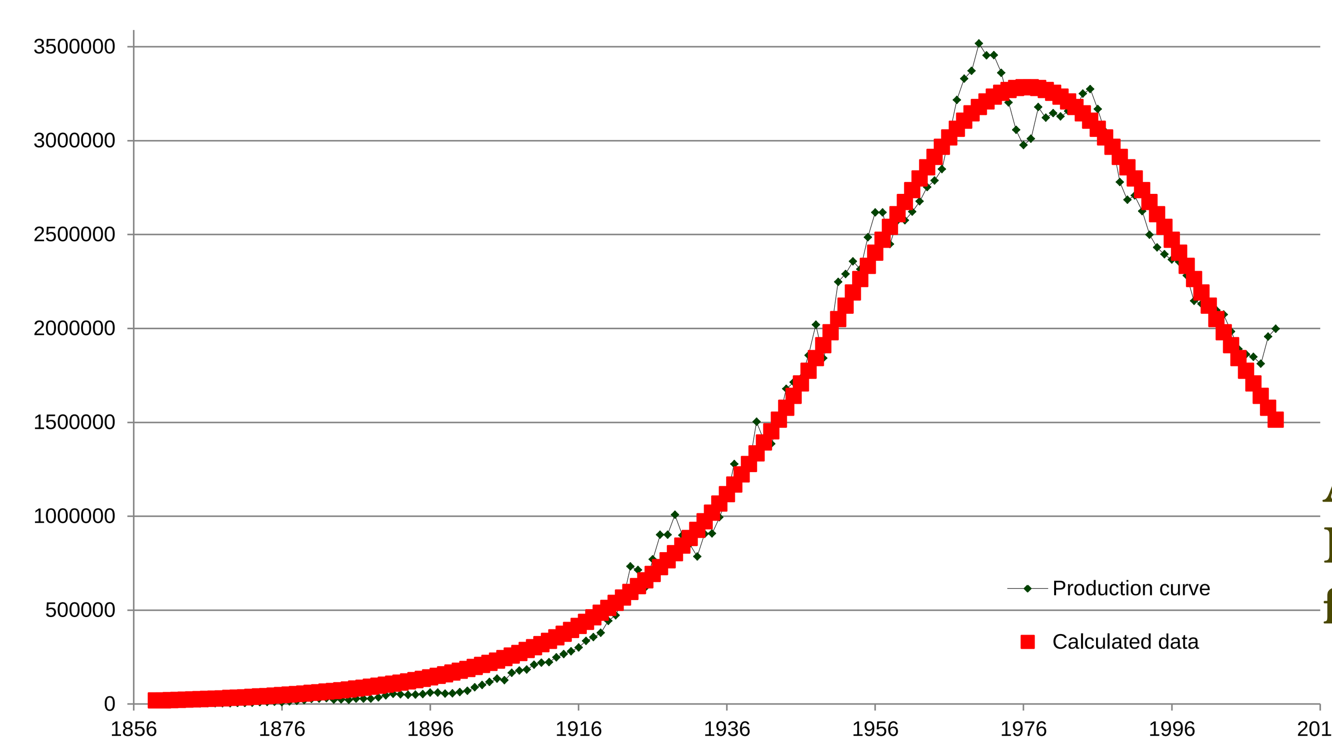


Mathematical Approach

Creating Formula

Because the production curve looks like the graph of an exponential function, the mathematical model often used is showed below where A, a, b, and c are constants, y is annual production in thousands of barrels per year and t is the year.

$$y = \frac{Ae^{-b(t-c)}}{(1+ae^{-b(t-c)})^2} \quad (1)$$



To graph the best representation of the production curve, SOLVER, an Excel Tool, is used to minimize the minimum deviation

$$\sigma = min\sum_{i=1}^{n}(y_{min} - y_i)^2$$

After step-by-step fitting the constants to Formula 1, the new equation is created with four ideal constants with A, a, b, c respectively:

$$y = \frac{3942039e^{-0.056(t-1998)}}{(1+0.3e^{-0.056(t-1998)})^2}$$

Differentiating

To find the year at which the production reached it peaks according to the model, differentiate formula (1) and set the derivative equal to 0 to find the concavity. Because the graph is non-negative, the concavity is the global maxima.

$$\frac{dy}{dt} = \frac{A(-b)e^{-b(t-c)} + Aabe^{-2b(t-c)}}{(1+ae^{-b(t-c)})^3}dt = \frac{e^{-b(t-c)}(Aabe^{-b(t-c)} - Ab)}{(1+ae^{-b(t-c)})^3}dt$$

$$\frac{dy}{dt} = 0 \Leftrightarrow e^{-b(t-c)}\left(Aabe^{-b(t-c)} - Ab\right) = 0 \quad \Rightarrow t = c + \frac{\ln a}{b} = 1977$$

With the calculations, the highest oil production in the U.S. was in 1977 of about 3,284,389 thousand barrels. Even though the production curve has two peaks, because of the new discovered oil reserves in Alaska in the early 1970s, the mathematical model is the graph of the exponential function which only has one peak in 1977.

Integrating

Mathematically, integrating the production curve from -∞ to ∞ would provide the total amount of petroleum at once available in the U.S.

$$\int_{-\infty}^{\infty}\frac{Ae^{-b(t-c)}}{(1+ae^{-b(t-c)})^2}dt = 234,645,179 \text{ (thousand barrels)}$$

Similarly, the total U.S. oil production up to 2010 were 203,149,451 thousand barrels. Subtract this number from the total amount gives the prediction of the undiscovered U.S. reserves of about 31 billions barrels.

Discussion

The calculation not only has the best result by minimize the standard deviation, its constants are also the best fit for this formula because it has shown the minimum summation of the partial derivative which are approximately to 0.

First, setting the residual at each data point t_i is

$$E_i = y_i - \frac{Ae^{-b(t_i-c)}}{(1+ae^{-b(t_i-c)})^2}$$

The sum of the square of the residuals is

$$S_r = \sum_{i=1}^{n}E^2{}_i = \sum_{i=1}^{n}(y_i - \frac{Ae^{-b(t_i-c)}}{(1+ae^{-b(t_i-c)})^2})^2$$

Then, differentiating by part with respect to each constant:

$$\frac{\partial S_r}{\partial A} = -1.14875\times10^{-36} \approx 0$$

$$\frac{\partial S_r}{\partial a} = 1.71124\times10^{-74} \approx 0$$

$$\frac{\partial S_r}{\partial b} = -8.4472\times10^{-27} \approx 0$$

$$\frac{\partial S_r}{\partial c} = 2.5359\times10^{-31} \approx 0$$

with

Symbol	Description	Units
y	U.S. oil production	Thousands of barrels/year
t	Time	Year
A=3942039	Constant	Thousands of Barrels
a=0.3	Constant	-
b=0.056	Constant	-
c=1998	Constant	-

References

Crude Oil Production. (2012, March 19)

U.S. Crude Oil, Natural Gas, and Natural Gas Liquids Reserves. (2010, November 30)

U.S. Energy Information Administration

CONCLUSION

As a result of the scarce and limited oil reserves in the US, the oil industry should find the most effective strategy to maintain the progression in the future. However, the total worldwide oil reserves are predicted that they are enough for the development and the high need of world populations in future decades. With the ultimate importance of petroleum resource, everyone should be informed about the scarcity of the finite resources in order to conserve the energy as much as possible. Saving the precious oil reserves not only ensures a long-lasting development of modern society, but also protects the environment more effectively.