Analyzing Ideal Blood Glucose and Insulin Levels

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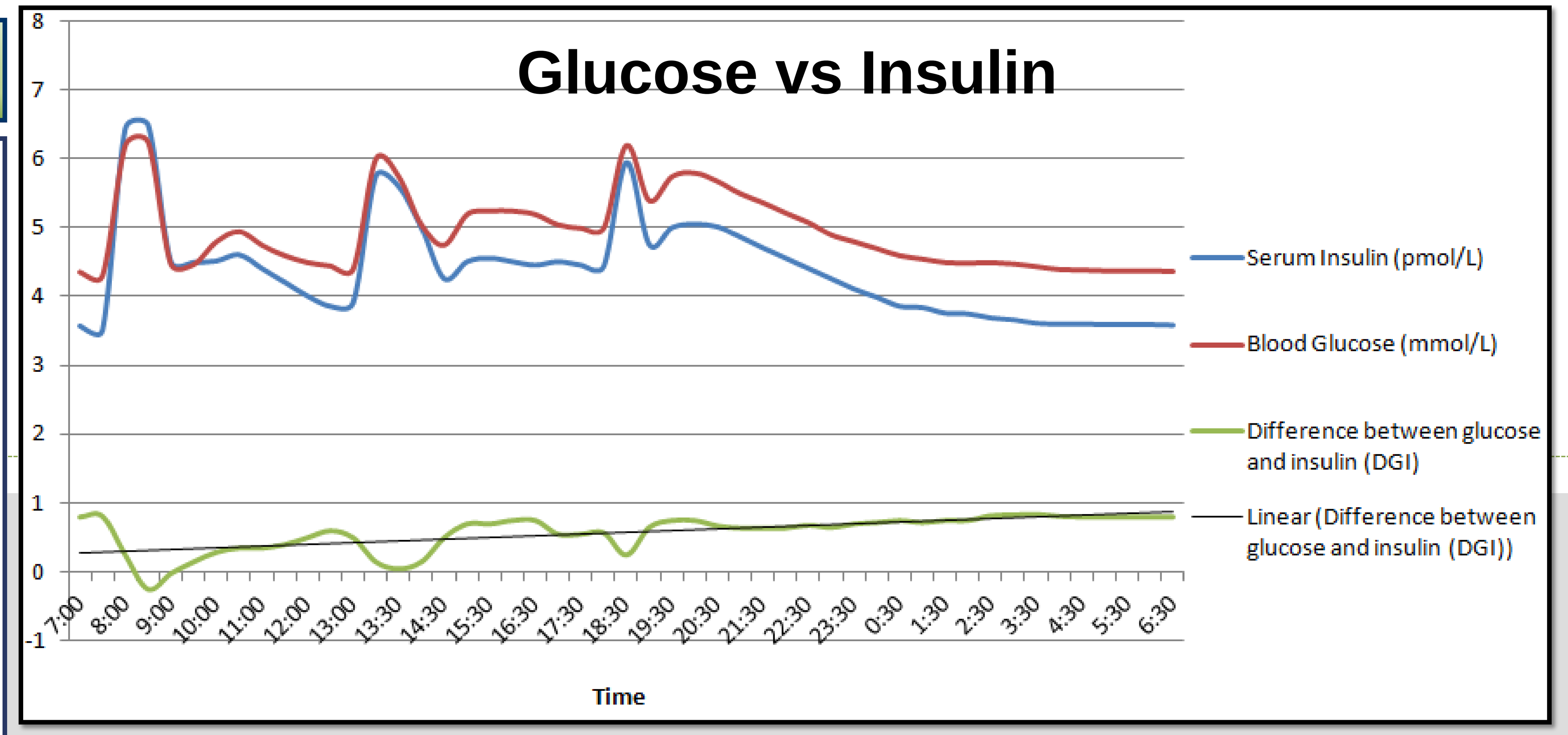
Abstract

Given the graph of the ideal values for Blood Glucose and Serum Insulin respectively, the value for Blood Glucose minus Serum Insulin was found and graphed. The graph of the difference (DGI) was then analyzed using several methods. First, the critical points were identified, tested, and found to be local minima that corresponded to breakfast, lunch, and dinner. Second, the time after dinner when the body enters a period of fasting was precisely determined by calculating the point of inflection at the parabola corresponding to dinner. Third, a linear function was found that could accurately describe the difference line during the over-night period of fasting. All of these trends have the potential to be used as diagnostic tools.

Motivation

In 2011, it was estimated that 25.8 million people in America have diabetes. [1] Many of these people rely on daily injections of insulin to control the levels of glucose (sugar) in their blood. Having to regulate your own blood sugar is a challenging and dangerous task. Too much glucose can cause a cardiac arrhythmia, painful tingling in the extremities, or even a coma. Not enough glucose can induce shaking, numbness, or tachycardia. Because of the severity of the disease, it is critical for doctors to recognize signs of diabetes in their patients.

Because diabetes can cause a deviation from normal blood glucose and insulin levels, afflicted individuals should be able to be identified by simple blood tests. With an estimated 79 million people in America categorized as pre-diabetic[1] , it is important for physicians and patients to have a good understanding of what normal blood glucose and insulin levels are so they can tell when something is wrong. Therefore, this report will analyze the ideal levels of blood glucose and insulin and recognize any trends that appear.



Mathematical Approach

Testing the local minimum of the difference line

In order to describe the normal relationship between blood glucose levels and insulin levels, the graph of the difference between the two values will be analyzed. The following critical points should be noted:

(x,y) = (8:30,-0.25),(13:15,0.05),(18:30,0.25) **(1)**

It can be observed by looking at the graph that (1) are local minima. Therefore,

$f^{I}(x,y) = 0$, $f^{II}(x,y) > 0$ at x = 8:30,
x = 13:15 and x = 18:30 **(2)**

To test one of the minima, one part of the step function was determined over the interval:

x {7:30,9:30}, **(3)**

At the interval (3), Microsoft Excel was used to find a formula that was approximately equal to the parabola. The equation was:

$f(x) = .1564x^2 – 1.0976x + 1.76$ **(4)**

Finding the first derivative of (4) gives us:

$f^{I}(x) = 0.3128x – 1.0976$ **(5)**

Finding (5) at zero will give us the critical point.

$f^{I}(x) = 0 = 0.3128x – 1.0976$ $f^{I}(3.51) = 0$ **(6)**

The critical point at (6) must be converted from units on a number line to the units on the graph of the DGI. The following shows this conversion.

3.51units * (30 min/1 unit) = 105.3 min = 1 hr 45 **(7)**

Setting the origin at 7:30 (the beginning of the interval), the critical point is precisely located at:

7:00 + 1hr45min = 8:45 **(8)**

Taking the derivative of (5) gives us the 2nd derivative of (4):

$f^{II}(x) = .3128$ **(9)**

Because (9) > 0 at the critical point (8), it is a local minimum.

Finding the point of inflection after dinner

One part of the step wise function of the DGI line is a parabola (that corresponds with dinner) between the intervals of:

x {18:00,19:30} **(10)**

A close approximation of the parabola was found in Excel. The equation is:

$f(x) = -0.17x^3 + 1.38x^2 – 3.27x + 2.62$ **(11)**

Finding the first derivative of (11) gives us:

$f^{I}(x) = -0.51x^2 + 2.76x – 3.27$ **(12)**

Finding the derivative of (12) gives us the 2nd derivative of (11),

$f^{II}(x) = -1.02x + 2.76$ **(13)**

To find the point of inflection for (11), we set (13) equal to zero.

$f^{II}(x) = 0 = -1.02x + 2.76$ $f^{II}(2.71) = 0$ **(14)**

To test the point of inflection, we will test units just before and just after,

$f^{II}(2.66) = -1.02x + 2.76$ $f^{II}(0) = .0468$
$f^{II}(2.76) = -1.02x + 2.76$ $f^{II}(0) = -0.0552$ **(15)**

According to (15), the function is concave up before 2.71 and concave down after. This is enough evidence to claim (14) as being a point of inflection. Because (14) is 2.71 units from the origin, we must apply this to the interval (10).

2.71 * (30 min/1) = 81.3 min = 1hr21min **(16)**

Assuming that the beginning (origin) of the parabola (10) is at 18:00, we can do the following:

18:00 + 1hr21min = 19:21 **(17)**

Describing the difference line as a linear function

The DGI line is closely approximated by a linear function along the following interval:

x {19:30,6:30} **(18)**

The values for anything along (18) can be approximated by the following function (found using Excel):

$f(x) = 0.0122x + 0.2689$ **(19)***

*To use the function (19), time (x) must be in units of hours. For example 21:30 would be equal to 21.5.

Discussion/Conclusion

- **Blood glucose was greater than insulin levels at every time except breakfast**

  This outlier in the data can be explained by the “dawn phenomenon”. Early in the morning, the body releases various hormones that increase blood sugar while diminishing insulin’s effects in order to prepare for waking up.[2]

- **The three local minima discussed in (2) directly correspond to three local maxima on both glucose and insulin lines. These maxima represent breakfast, lunch and dinner.**

- **That the equation (19) was highly accurate in predicting the value of the difference line between the intervals in (18) that correspond to the fasting stage.**

- **The point of inflection after dinner corresponds to the beginning of the fasting state.**

In this basic analysis of the data, the values of the difference between blood glucose and serum insulin levels proved to display trends that directly correlated to healthy metabolic regulation. Further studies should be conducted to find out what average blood sugar and insulin levels would look like for type 1 and type 2 diabetes patients throughout the day. These samples can then be compared against the ideal level trends. If common deviations occur between the different samples then they can be used as models to predict the health condition of other patients. **More accurate diagnoses lead to more effective treatments.**

References

[1] - "Diabetes Statistics." American Diabetes Association. N.p., 26 Jan. 2011. Web. 3 May 2012. <http://www.diabetes.org/diabetes-basics/diabetes-statistics/>.

[2] – "Diabetes Health Center." WebMD. Ed. John A. Seibel MD. N.p., 25 Feb. 2010. Web. 3 May 2012. <http://diabetes.webmd.com/morning-high-blood-sugar-levels>.

[3] - Suckale, Jakob, and Michele Solimena. "Pancreas Islets in Metabolic Signaling - focus on the beta-cell." Nature Precedings 9 June (2008): 3-4. Web. 3 May 2012. <http://precedings.nature.com/documents/1724/version/2/files/npre20081724-2.pdf>.