Utilization of Calculus in Light Attenuation

“Average Light Intensity Inside a Photobioreactor”

Herby Jean

herbyj@mail.usf.edu

University of South Florida, Tampa, FL

Arcadii Grinshpan

Mathematics and Statistics

Daniel Yeh

Civil & Environmental Engineering

Abstract

For energy production, microalgae are one of the few alternatives with high potential. Similar to plants, algae require energy acquired from light sources to grow. This project uses calculus to determine the light intensity inside of a photobioreactor filled with algae. Under preset conditions along with estimated values, we apply Lambert-Beer’s law to formulate an equation to calculate how much light intensity escapes the photobioreactor and determine the average light intensity that is present inside the photobioreactor at any given time.

Problem Statement

After researching the potential use of algae as a renewable resource for producing fuel, we concluded that the energy for algae to grow is essential in order to control algae production in labs and other facilities. Given an algae concentration of 1 g/L in a photobioreactor with length 0.05 m, and sunlight supplying light with an intensity of 1000 mmol/m²s, calculate the light intensity inside the photobioreactor and the amount of light that leaves the reactor.

Mathematical Approach

We begin by examining Figure 1; the concept of light attenuation is used to determine how fast light intensity decreases with distance within an object. The figure illustrates the effect on light intensity when the light comes in contact with a reactor or in our case a photobioreactor. Initially, the light strikes the reactor with a constant intensity but when the light is inside the reactor it is converted into photonic energy fueling the algae concentration to grow. As the light continues to move through the reactor the intensity decreases and when the light passes fully through the photobioreactor less light exits the photo bioreactor than initially entered. The question remains as to what happens with the light intensity as it passes through the photobioreactor and how could we calculate such a phenomenon.

We apply Lambert-Beer’s law which states that

$$I_{out} = I_{in} \cdot e^{-a_c c_{alg} b} \qquad (1)$$

where I_{in} (I_{out}) is the light intensity entering (exiting) the photobioreactor, a_c is the spectral-averaged absorption coefficient, c_{alg} is algae concentration, and b is the length of the photobioreactor.

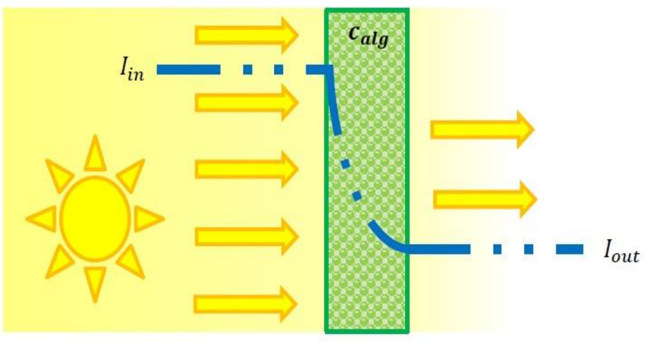

Figure 1: Light attenuation in a photobioreactor.

We let a_c=200 m²/kg and assume the initial conditions , I_{in}= 1000 mmol/m²s, c_{alg}=1 g/L, and b=0.05 m. Substituting these parameters into equation (1) yields,

$$I_{out} = 1000\, e^{(-200)\cdot(1)\cdot(0.05)} = 0.0454 \text{ mmol/m}^2\text{s} \qquad (2)$$

Applying Lambert-Beer’s formula once again, this time over the length of the reactor, we calculate the average light intensity to be

$$I_{ave} = \frac{\int_0^b I_{in}\, e^{-a_c c_{alg} x} dx}{\int_0^b dx} = \frac{I_{in}\left(1 - e^{-a_c c_{alg} b}\right)}{b\, a_c\, c_{alg}} = 99.99 \frac{\text{mmol}}{\text{m}^2\text{s}} \qquad (3)$$

Discussion

Symbol	Description	Units
a_c	Spectral-averaged absorption coefficient	m²/kg
b	Length of photobioreactor	m
C_{alg}	Concentration of algae	g/L
I_{in}	Light Intensity entering photobioreactor	mmol/m²s
I_{out}	Light Intensity leaving photobioreactor	mmol/m²s
I_{ave}	Average light intensity inside photobioreactor	mmol/m²s

Lambert-Beer’s Law became crucial in deriving an equation to find the average light intensity inside the photobioreactor. Inside the photobioreactor, the average intensity of sunlight came out to be 100 mmol/m²s. Also, Lambert-Beer’s law allowed us to calculate the light intensity leaving the reactor, 0.0454 mmol/m²s . We predicted the amount of light intensity leaving the reactor would be less than the amount of light intensity entering the reactor. However, we did not expect the result to be as low as we calculated. Our calculations suggest almost all of the light intensity from sunlight was taken and used as photonic energy by the photo bioreactor. These results raised the possibility of increasing or decreasing the length of the photobioreactor. How much of a difference would a reactor with a path length of 0.05 m be in comparison with a reactor of 0.02 m? In algae production, the size of the reactor needs to be determined carefully in order to receive the maximum input of the light source. The concentration of algae will also determine how much light intensity leaves the reactor. In this project, we assumed the sunlight was supplying the reactor with 1,000 mmol/m²s in light intensity. In future applications, the outcome and the light source may be different. Aside from sunlight radiation, there are various of human-made light sources available. From everyday incandescent lights to high pressure sodium lights in Figure 2, each light holds an adequate amount of light intensity to aid in algae growth inside the reactor.

Figure 2: Top left to right: Halogen light, Fluorescent light
Bottom left to right: Incandescent light, High Pressure Sodium light.

Conclusions

With the presence of algae inside the photobioreactor the conclusions we derived regarding light intensity are straightforward: the light intensity entering the reactor was 1000 mmol/m²s, the light intensity inside the reactor averaged out to 100 mmol/m²s , and light intensity leaving the reactor was calculated to be 0.0454 mmol/m²s. These results illustrate the use of a particular light source by the algae concentration; we notice nearly all of the light intensity was used by the algae as less than 1% of the intensity of the light entering the reactor exited the reactor. The amount of algae growth was not calculated in this project and could be the topic of future research. As a continuation of this project, the study of growth rate of algae as function of light intensity could prove to be beneficial in understanding the dynamics of algae organisms.

References

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