THE ELEVATION TO AREA RELATIONSHIP OF LAKE BEHNKE

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Abstract

Our objective was to determine the area-to-depth relationship in Lake Behnke, which acts as the principal stormwater drainage basin for the University of South Florida campus in Tampa, Florida. Previously collected data from a stormwater management study illustrated a linear correlation between the lake's depth and the area at the corresponding depth for a sampled part of the lake. This project expanded on the previous study by examining the entire lake, correlating the depth at two foot intervals with the respective areas, in order to verify the prior results. With a bathymetric map of Lake Behnke that displayed several contour lines indicating depth, the area inside each closed curve was determined using an approximation of the area given by the contour integral. This was done by partitioning the closed curve into many subintervals, a method similar to that of the Riemann sums. The resulting relationship between the area and elevation from the rest of the lake's depth was determined to be parabolic.

Problem Statement

Given the bathymetric map of Lake Behnke, determine the areas that correspond to each elevation, and evaluate how this relationship compares to a previous study conducted on the lake.

Motivation

Lake Behnke and Hypsographic Curves

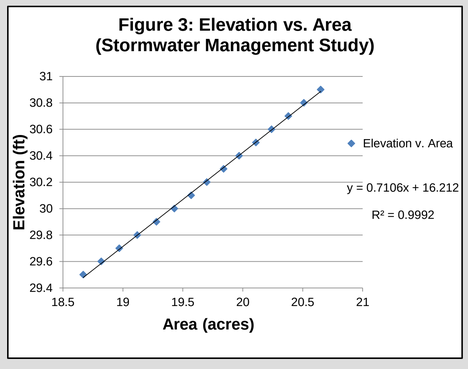
- Stormwater must be retained in Lake Behnke in order to meet the county's "pre-development vs. post-development volume requirement."
- Retaining this excess water is also of ecological importance.
- A hypsographic curve for a lake is the visual representation of the relationship between elevation and area, and is extremely important in predicting the surface areas of a lake when the water level changes.
- It would be very useful to establish a hypsographic curve for Lake Behnke.

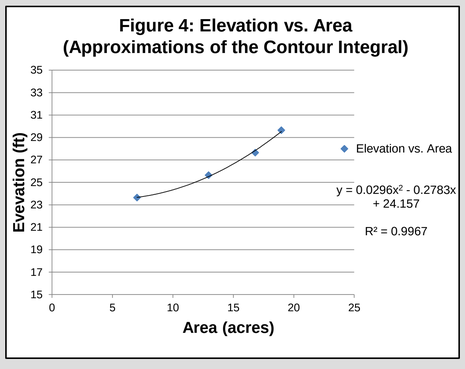
1998 Stormwater Management Study

- The USF Master Stormwater Management Study collected data on the lake fourteen years ago, and sampled the areas for the first foot and a half of the lake's depth (Fig. 3).
- It demonstrated a linear relationship between the depth and area of Lake Behnke.
- The current project aimed to expand the analysis to the *entire* lake, and verify the relationship previously documented in the Master Stormwater Management Study.

Figure 1: Lake Behnke

Figure 2: Bathymetric Map of Lake Behnke

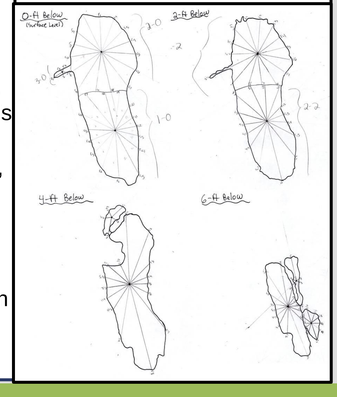




Mathematical Approach

- Using a bathymetric map, which illustrates underwater topography (Fig. 2), the contour lines at each 2ft difference in elevation were traced to produce four closed curves (Fig. 5).
- **The areas at each depth were estimated by an approximation of the area given by the contour integral.**
- Each simple closed curve can be partitioned into ***n*** subintervals, where each partition has an angle $\Delta\theta_k$ and the sum of all angles is equal to 2π.
- A particular length, from the center point to the side of the curve, corresponds to an angle at the center and is denoted $\rho(\theta_k)$; that is, the length as a function of the angle.
- The contour integral $\frac{1}{2}\int_c \rho(\theta)^2 d\theta$ gives the area within each closed curve
- The contour integral can be approximated by $\frac{1}{2}\sum_{k=1}^{n} \rho(\theta_k)^2 \Delta\theta_k$ when ***n*** approaches infinity and the maximum $\Delta\theta_k$ value (where $1 \le k \le n$) approaches zero.

Figure 5: Approximation of Area at 2 ft intervals



Discussion

- The resulting areas from the approximations of the contour integral are shown graphically in Figure 4, and illustrate a hypsographic curve for the whole of Lake Behnke that is more parabolic than linear.
- These results fit reasonably well with the data from the 1998 Stormwater Management Study, but indicate a decrease of ~1ft in surface elevation and ~2 acres in area from 1998 to 2011.
- This slight decrease in surface elevation and area can be explained by:

1) Drought: since Lake Behnke collects stormwater, a lack of precipitation will result in decreased water levels.
2) Sedimentation: because Lake Behnke collects run-off, it also traps all of the sediment contained in that water. Sediment "filling in" the sides and the bottom of the lake could account for the reduction in area.
3) Karst Processes: Lake Behnke is a natural sinkhole feature and therefore has a layer of soluble bedrock that is easily dissoluted by chemical processes, causing the lake atop it to slowly sink lower.

Conclusions

- **From this project, it can be concluded that the relationship between elevation and area documented in the 1998 Master Stormwater Management study is NOT representative of the entire lake.**
- **The slight differences in surface elevation and areas can be attributed to a combination of weather (drought), sediment loading, and geological processes of the surrounding land.**
- Since Lake Behnke is the main drainage basin for half of the USF campus, it is very important to know its physical capacity to hold water and the maximum surface area it can contain without exceeding the post-development outflow regulations.
- It is also important in that a great concentration of pollutants may end up in the lake, and having a different depth-to-area relationship may change the lake's capacity to dilute these pollutants

References

1) A Beginner's Guide to Water Management - Lake Morphometry (Inf. Circ. 104). Florida LAKEWATCH, Department of Fisheries and Aquatic Sciences, Institute of Food and Agricultural Sciences, University of Florida IFAS. September 2001, 2nd Edition.
2) Clemson University. "Stormwater Pond Design, Construction, and Sedimentation." *Clemson University Cooperative Extension*. Clemson University, 2012. Web. 1 May 2012.
3) Earhart, Jeffrey J, engineer. Final Future Condition Analysis: University of South Florida Master Stormwater Management Study. Rep. Orlando, FL: Dyer, Riddle, Mills, and Precourt, (Engineers, Surveyors, Planners), 1998. Print.
4) Foley. "Water Quality Analysis." *Introduction to Environmental Science Laboratory Manual*. Tampa: University of South Florida, 1993. 91, 105.
5) Florida Center for Community Design and Research, GIS Team Water, Atlas Program. "Lake Behnke." Map. *Hillsborough County & City of Tampa Water Atlas*. WaterAtlas.org, 2001-2012. Web. 1 May 2012.
6) "What Is Karst?" *University of Texas at Austin*. Environmental Science Institute, University of Texas. Web. 15 Nov. 2011. <http://www.esi.utexas.edu/outreach/caves/karst.php>.