Solar Insolation

Modeling Solar Insolation with Integration

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Abstract

With interests in energy resource concerns, we pursued modeling solar insolation. We calculated the amount of energy a window in the northern hemisphere facing due south may receive at different latitudes and different days of the year by manipulation of sun position equations. This model stands to highlight how thoughtful design can maximize or minimize entering sunlight over different seasons, saving on heating and cooling, respectively.

Problem Statement

Implementing methods to minimize use of nonrenewable energy sources is imperative. One can model buildings and then, using concepts from Calculus, determine methods to construct that maximize free resources, such as sunlight. Utilizing known angles of the sun (zenith angles and solar altitude angles) at different times of the year along with global positioning, it is possible to decipher where to best orient walls and windows of a home.

Mathematical Approach

We wish to use three-dimensional modeling in conjunction with solar angles and the formula for solar insolation: I = S coz (Z), to develop the best orientation for a home at a given geographic location (Stickler).

I = Solar Insolation
S = Solar Constant (1000 watts/m²)
Z = Zenith Angle (Dependent on latitude, solar declination, and time of day)

First, we rearranged to spherical coordinates. The resultant integral follows:

$$\int_{sunset}^{sunrise} S\cos M dt = \int_{sunset}^{sunrise} S\sin Z_1\left[\cos\left(Azm_1 - Azm_2\right)\right]dt$$

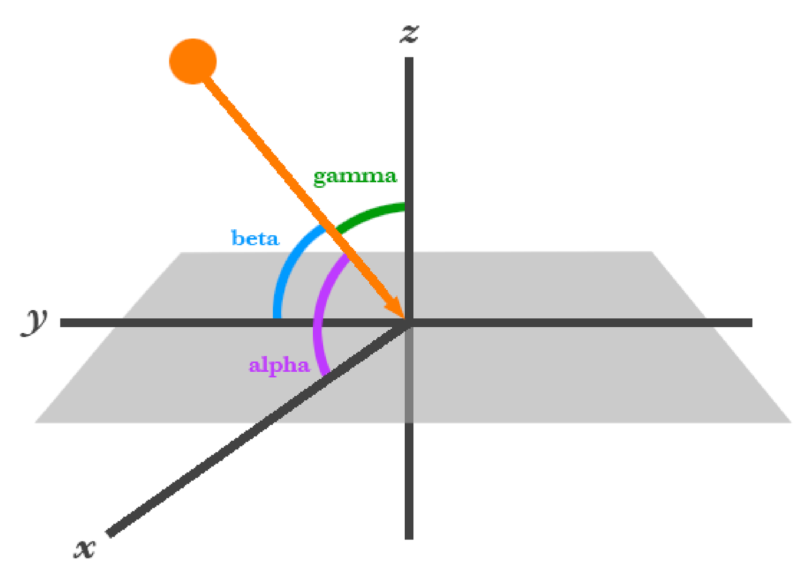
$$Z = \arccos\left\lfloor \sin\phi\sin\delta + \cos\phi\cos\delta\cos\left(15\left(T-12\right)\right)\right\rfloor$$

$$Alt = \arccos\left\lfloor \sin\phi\sin\delta + \cos\phi\cos\delta\cos\left(15\left(T-12\right)\right)\right\rfloor$$

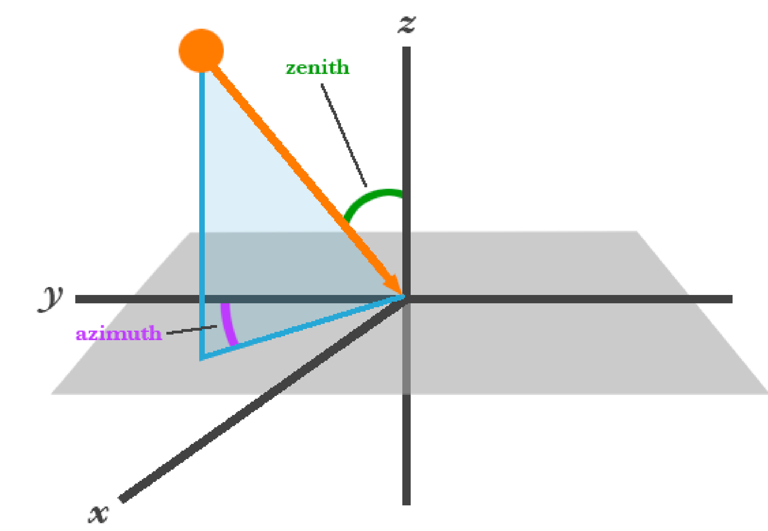
$$Azm = \arcsin\left[\frac{\cos\delta\sin\left(15\left(T-12\right)\right)}{\cos\left(Alt\right)}\right]$$

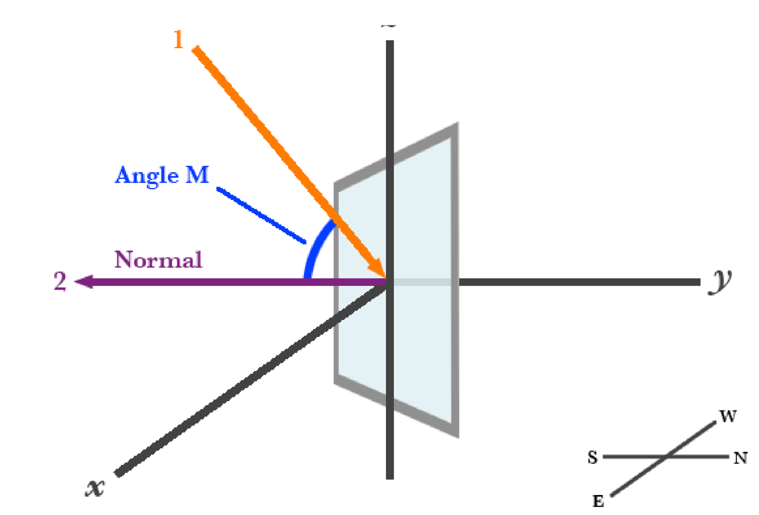
Final Integral =

$$\int_{sunset}^{sunrise} S\sin\left\{\arccos\left[\sin\phi\sin\delta + \cos\phi\cos\delta\cos\left(15\left(T-12\right)\right)\right]\right\} * \cos\left[\frac{\cos\delta\sin\left(15\left(T-12\right)\right)}{\arcsin\left[\sin\phi\sin\delta + \cos\phi\cos\delta\cos\left(15\left(\left(T-12\right)\right)\right)\right]} - 90\right]dt$$

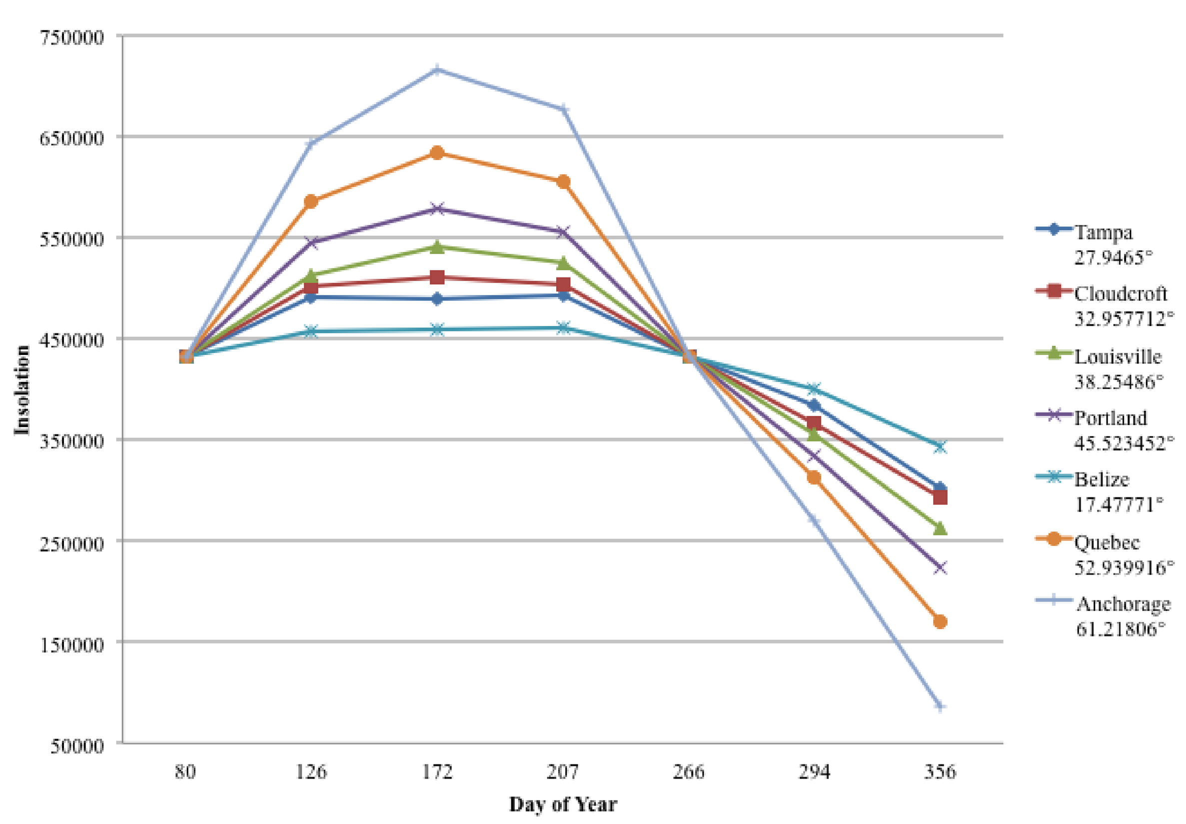


Discussion





Variation in solar radiation resulted as expected. When evaluating direct solar radiation (perpendicular to a location) one should expect different results and higher annual insolation values for places closer to the equator.



Conclusions

In this evaluation we did not address the amount of solar radiation that may deflect in the atmosphere. We intended to give rough estimates and to demonstrate how useful calculus analysis can be in the design process, to save precious energy resources in any closed structure. Creative design concepts are worth investigating whether building something new or working to improve existing property.

References

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